CAS Lecture Notes Number 1

Center for Advanced Studies, Warsaw University of Technology
Warsaw

Feng Gao and Jialun Yang

Topology synthesis for parallel robotic mechanisms

Prof. Dr. Feng Gao
State Key Laboratory of Mechanical System and Vibration
School of Mechanical Engineering
Shanghai Jiao Tong University

Dr. Jialun Yang
China Electric Power Research Institute

Editor: **Stanisław Janeczko**

Technical editors: **Ewa Stefaniak, Małgorzata Zielińska, Anna Żubrowska**

General layout and cover design: **Emilia Bojańczyk / Podpunkt**

For additional information on this series please visit the CAS Publications Website on www.csz.pw.edu.pl

ISBN: 978-83-61993-09-4
Printed in Poland

Contents

Preface

Mechanisms are the core of mechanical products and mechanism design determines the level of innovation. The study of modern mechanisms is very important in order to increase the level of mechanism research and to raise the international competence capability of mechanical products. Compared to the serial structure, parallel one has its own characteristics, such as a big payload, compact structure and a coupled relationship between input and output. Consequently, parallel mechanisms have more and more potential applications. Furthermore, parallel robotic mechanisms are complicated systems, and the design of such is the core theory and technology base of modern complicated equipment.

In the robotic field, parallel mechanisms have been a key research topic which includes analysis and synthesis. The synthesis of parallel robots mainly comprises type synthesis and dimension synthesis. For type synthesis of parallel robots this book studies the three main problem areas which cover such topics as how to establish the performance criteria for the type of robots, whether there are any laws and algorithms for the type synthesis of robotic mechanisms and how to synthesize the desired parallel mechanisms according to the laws and algorithms, respectively. The main contents include: the definition, description and classification of the G_F sets; the laws for translations and rotations, and the three related corollaries; the relationships between the number of limbs, number of actuated limbs, dimension of the characteristics of end-effectors of parallel topologies, number of limb actuators, and the number of passive limbs; the properties and algorithms of intersections of the G_F sets; the axis movement theorem and its application for the design of limbs with specific characteristics of end-effectors; the compositions of characters of end-effectors of topologies and corresponding conditions; the type of parallel mechanisms with pure translation, one-dimensional rotation, two-dimensional rotations, and three-dimensional rotations.

The authors hope that the book will bring new ideas to both the topological approach in the design of parallel structures and to the knowledge system of modern mechanisms, which may lead to a more practical application of parallel mechanisms in the near future.

Acknowledgments

This research is supported by the National Basic Research Program of China (973 Program), Natural Science Foundation of China (NSFC).

1. Introduction

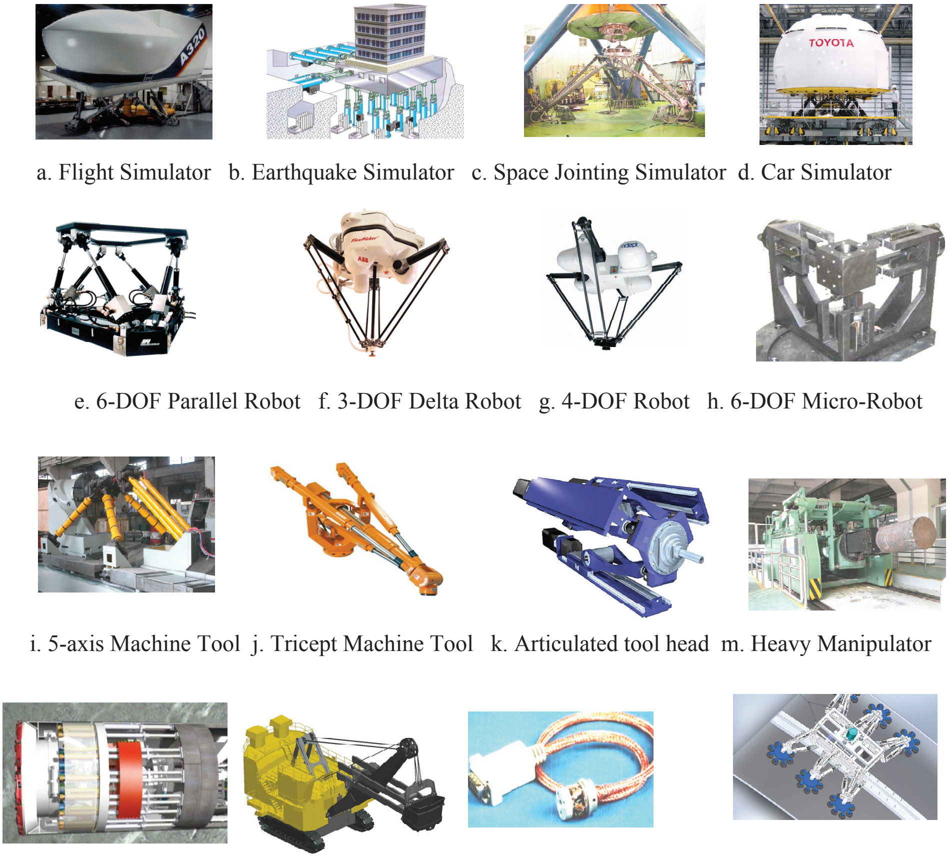
1.1. Applications of parallel mechanisms

Design is the thought process comprising the creation of an entity, and is one of the highest-level human endeavors that demonstrates the intelligence, wisdom and creativity of human beings. The design process can be generally summarized as three main steps: to propose clearly defined objectives, to establish design algorithms, and to search for the solutions of these desired objectives by using algorithms. Mechanical design includes two main problem areas, that is, the design of mechanisms and the design of structures. For the design of mechanisms there are three important issues which are: performance criteria for evaluating characteristics, type synthesis and dimension synthesis of mechanisms.

Although mechanisms research has a very long history, parallel mechanisms have become a crucial topic in the international robotic research area since Hunt defined the Stewart platform as a parallel robot in 1978. The characteristics of parallel robots are high stiffness, high payload, compact structure and low inertia, and they have potential application in the field of multi-dimensional heavy manipulation, micro manipulation and dynamic simulation, such as space docking prototypes, flight training devices, vibration and seismic simulators, mining products, tunnel shields, forging manipulators and minimal-invasive surgery devices, etc., as is shown in Fig. 1. The challenge of designing parallel robots lies in the fact that the topological structures cannot be described by general algebra, the performance evaluation should be numeric and the dimension synthesis is a nonlinear multi-objective issue. Consequently, the design theory for parallel robots has long been a current topic in the international robotic area.

1.2. Analysis on type synthesis of parallel mechanisms

Because topologies express the most fundamental performances of robotic mechanisms, the invention of new structures is a challenging and original issue in research on robotic mechanisms. Recent advances in type synthesis of robotic mechanisms have led to an explosion in the number of mechanism types with multiple degrees of freedom. There are several kinds of mathematical tools for



a. Flight Simulator b. Earthquake Simulator c. Space Jointing Simulator d. Car Simulator

e. 6-DOF Parallel Robot f. 3-DOF Delta Robot g. 4-DOF Robot h. 6-DOF Micro-Robot

i. 5-axis Machine Tool j. Tricept Machine Tool k. Articulated tool head m. Heavy Manipulator

n. Tunnel Borer o. Excavator p. 6-Axis F/T Sensor q. Climbing-drilling Robot

Fig. 1. Applications of parallel robotic mechanisms

the topological design of parallel robotic mechanisms: DOF equations [1, 2], graph theory [3, 4], group theory [5, 6, 7], screw theory [8, 9, 10, 11], linear transformation [12], POC equations [13], set theory [14, 15, 16], etc. By utilizing the above-mentioned methodologies, many novel parallel mechanisms have been proposed and several of them have been commercialized. The use of applications of parallel robotic mechanisms has increased dramatically. Many researchers such as, Tsai, L.W., Earl, C.F., Hervå?, J.M., Li, Z.X, Huang, Z., Li, Q., Kong, X., Gosselin, C.M., Gogu, G., and Yang, T.L. and others, have played key roles and made important contributions in the field of type synthesis of parallel robotic mechanisms. Despite all the advances in this area which have led to many successful applications [17, 18, 19], the question remains as to whether there exist certain basic algorithms for the analysis of kinematic mobility which can be used to design the topologies of parallel mechanisms. Researchers have proposed the DOF equations, graph theory, group theory, screw theory, linear transformation, POC equations, and set theory to synthesize the types of parallel robotic mechanisms. The topologies of the mechanisms are non-algebraic, dimensionless and independent of the choice of coordinate systems. Therefore, it is necessary to summarize the type synthesis methods as follows.

"**The DOF Equations**" concern the number of inputs of the mechanisms which are usually used for the "number synthesis of mechanisms". Number synthesis is used to investigate the relationships among the degrees of freedom, links and kinematic pairs. The degrees of freedom may lead to some confusion because the DOF equations can only be used to calculate the input number of mechanisms and cannot be employed to analyze the mobility of the end-effector of robots. For type design of parallel robotic mechanisms we mainly pay attention to the properties of the end-effectors of the robots, therefore, the DOF equations cannot be used to analyze the relationships among the number of limbs, number of actuated limbs, dimension of characteristics of the end-effectors for parallel topologies, number of actuators of the limb, and the number of passive limbs for parallel robotic mechanisms.

"**The Graph Theory**" is the mathematical tool that investigates the relationship between the "Edges" and "Vertices" of the "Graph". In Graph Theory, by using the DOF equations to get the number synthesis results (input number, link number and kinematic pair number), the kinematic pairs of the mechanisms are changed into the "Edges", and the links are changed into the "Vertices", so it is suitable for type synthesis of planar mechanisms and difficult to be utilized for type synthesis of parallel robotic mechanisms.

In "**Screw Theory**" there are no intersection rules for screw motions, so that the union rules of force constraint screws have to be used for type synthesis of parallel robotic mechanisms. The force constraint screws can be obtained by the reciprocal screws of the motion screws, but the reciprocal screws do not match the motion screws one by one. Because two kinds of two-dimensional rotation screws may correspond to one force constraint screw, the union rules of force constraint screws used for type synthesis of parallel robotic mechanisms sometimes may result in confusion or may lead to mistakes. In addition, the screw coordinate is instantaneous, so the full cycle mobility should be checked for the synthesized mechanisms.

For the "**Group Theory**", only part of the motion types can be dealt with because there are no Lie sub-groups with two rotations or the dimensions of the rotational components are larger than the dimensions of the translational components. Most of the mathematical tools used for synthesizing parallel mechanism types are related to both the algebra and choice of coordinate systems, for example, the linear transformation method or the method based on POC equations.

1.3. Problems of type design of parallel robotic mechanisms

A design process can generally be summarized into three main steps. These are: to propose clearly defined objectives, to establish design algorithms, and to search for solutions for the desired objectives by using design algorithms. Although many kinds of methods for type synthesis of parallel robots have been proposed, there are still problems with the type synthesis of parallel robots. These are as follows:

- What kind of mathematical method is suitable for type synthesis of robots?
- What kind of performance criteria can be used for describing and evaluating the characteristics of the end-effectors of robots which are not in algebraic form, without a unit, or not related to the coordinate systems?
- What kind of basic laws for kinematic mobility can be utilized to establish the computing rules of intersection for the topology design of robotic mechanisms?
- How can the model of number synthesis which shows the relationships among the number of limbs, number of actuated limbs, dimension of characteristics of the end-effectors, number of actuators of the limb, and number of passive limbs for parallel robotic mechanisms be obtained?
- How should the limbs with specific output performance be designed?
- How should the solutions for the topologies of robotic mechanisms based on performance criteria and laws be searched?

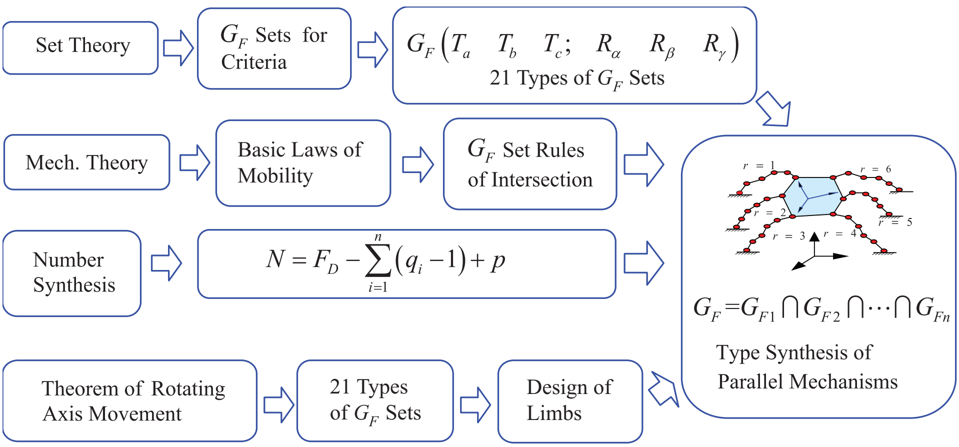


Fig. 2. Type design procedures of parallel robotic mechanisms

To solve these problems we propose the G_F set for type synthesis of parallel robotic mechanisms. The type design procedures of parallel robotic mechanisms as shown in Fig. 2 as follows:

1. According to the requirements of the application and 21 types of G_F sets, to give the G_F set for the robotic mechanisms;
2. Using the number synthesis model, to search the results of number synthesis;
3. From the G_F set rules of intersection, to select G_F sets of all the limbs;
4. Utilizing theorems of rotating axis movement, to design all the limbs;
5. Synthesizing topologies of parallel robotic mechanisms.

2. G_F Sets and their Characteristics

2.1. Definition and classification of G_F sets

Many criteria have been established to evaluate the performance of robotic mechanisms, for example, the workspace, velocity, acceleration, payload, stiffness, etc., all of which are in algebraic form with unit, related to the coordinate systems and can not handle the extremely diverse range of topology performances of robotic mechanisms. We must develop an efficient and general performance criterion that is applicable to the design of the topologies for robotic mechanisms which should be non-algebraic, dimensionless and independent of the choice of coordinate systems. As a result, we propose special sets to describe the topological performance property of the kinematic mobility of the end-effectors of robotic mechanisms. The sets are named **generalized function sets** (G_F sets for short), that is, $G_F(T_a T_b T_c; R_\alpha R_\beta R_\gamma)$, where the components $T_i, i = a, b, c$ denote the translational characteristics of the end-effector, and the components $R_j, j = \alpha, \beta, \gamma$ which represent the rotational characteristics of the end-effector described in terms of three successive rotations. As shown in Fig. 3, the axes of these three rotations are denoted by R_α, R_β, and R_γ, respectively, where R_α is the base rotation axis, R_β is the middle rotation axis relative to R_α, and R_γ is the last rotation axis relative to R_β. Successive rotations are used because in the natural world there is no pure rotation about two fixed axes. Here, the term "pure" means there are no accompanying translations and rotations. Consequently, when the element is non-zero, then the corresponding capability exists; when the element is zero, then the corresponding capability does not exist. As different points of an end-effector have different characteristics with respect to the rotations, it is important to mention the specific point of the end-effectors here. From Fig. 3 we can see that the characteristics of point P on the end-effector include both the translational and rotational components. In the upper part of Fig. 3b, the term "TE" means translational components. The components T_a, T_b, and T_c are non-coplanar simultaneously, and two of them are not collinear. In the lower part of Fig. 3b, the term "RE" refers to rotational components. The