

CAS Lecture Notes Number 8

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**On the mathematical modeling
of complex systems**

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Preface

These lectures are based on a cycle of four presentations that were delivered at Warsaw University of Technology in November and December 2012 under the auspices of the Center for Advanced Studies. The intention is to give an introduction to some recent mathematical approaches to the modeling of complex systems. Complex systems are characterized by their functioning on various levels that differ by more than just the scale of the parameters. The basic modeling philosophy is top-down rather than bottom-up, focusing on what an observer knows (or cares to know) about the system, as opposed to striving for the kind of complete, reductionist analysis which is the more usual goal of traditional applied mathematics.

The first lecture is designed to give an overview of the approach, which may be read independently of the subsequent lectures. Two key models are discussed: both a theoretical model of competition and a practical model of demography. These models are treated in greater depth in the second and third lectures. The fourth lecture covers the new mathematical topics, such as modal theory and hierarchical statistical mechanics, which are being developed to support the study of complex systems.

