Mathematical Physics of Hurwitz Numbers:
Stratification of Hurwitz Spaces and Singularities

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Hurwitz numbers enumerate ramification coverings of the Riemann sphere with a fixed configuration of ramification points and ramification types over these points. Equivalently, they enumerate all possible factorizations of a permutation of given cyclic type into a product of a given number of premutations of given cyclic types. Being a rather elementary combinatorial object, they are related to various domains in modern mathematics and mathematical physics: moduli spaces, Gromov-Witten theory integrable systems, and so on. Thus they could serve as a convenient elementary model for the study of all these domains.

3. Stratification of Hurwitz Spaces and Singularities

The geometric counterpart of the Hurwitz numbers are Hurwitz spaces which are moduli spaces of meromorphic functions on curves of fixed genus. The Hurwitz spaces are stratified according to possible degeneracy of critical points of the function. The local study of singularities of this stratification provides also an information on the global topological properties of the strata. We show how the ideas of Thom polynomials in the global singularity theory allows one to convert the obtained information on the stratification of the Hurwitz spaces into explicit formulas for families of Hurwitz numbers.