Mathematical Physics of Hurwitz Numbers:
Hurwitz Numbers and Integrable Systems
Prof. Maxim Kazaryan

Hurwitz numbers enumerate ramification coverings of the Riemann sphere with a fixed configuration of ramification points and ramification types over these points. Equivalently, they enumerate all possible factorizations of a permutation of given cyclic type into a product of a given number of premutations of given cyclic types. Being a rather elementary combinatorial object, they are related to various domains in modern mathematics and mathematical physics: moduli spaces, Gromov-Witten theory integrable systems, and so on. Thus they could serve as a convenient elementary model for the study of all these domains.

1. Hurwitz Numbers and Integrable Systems

We show that properly organized generating function for Hurwitz numbers possess nice integrable properties, in particular, it is a solution to the KP (Kadomtsev-Petciashvili) integrable hierarchy of PDEs. We describe the geometry of these equations and explain the machinery producing their solutions.